17: Is $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$? (because it *looks* right?)

We know that the right hand side, $\frac{1}{a+b}$, cannot be equal to the left hand side, $\frac{1}{a}+\frac{1}{b}$, because, from Misconception 12, we know that

$$\frac{n}{a}+\frac{m}{b}=\frac{n\times b+m\times a}{a\times b}$$

The $\frac{1}{a}+\frac{1}{b}$ here is like the example above, with 1s in place of the n and m, so it equals

$$\frac{1\times b+1\times a}{a\times b}$$

that is

$$\frac{b+a}{a\times b},$$

which is nothing like $\frac{1}{a+b}$!

We could deduce that $\frac{1}{a+b}$ cannot equal $\frac{1}{a}+\frac{1}{b}$ by thinking of what things mean. Start with $\frac{1}{a}$

Changing this into

$$\frac{1}{a+b}$$

means increasing the divider, thus making the result smaller than $\frac{1}{a}$;

On the other hand,

$$\frac{1}{a}+\frac{1}{b}$$

is bigger than

$$\frac{1}{a},$$

so $\frac{1}{a+b}$ cannot be equal to $\frac{1}{a}+\frac{1}{b}$.