21: Definite Way to Deal with Probability

Toss two coins – what is the probability of getting a head and a tail?

Firstly, beware of intuition. Often, intuition not only fails to give the right answer, it suggests a wrong answer!

The Method:

Make a comprehensive list of all the possible outcomes, and count them, (call this sum '*t*' -for 'total')

Next, count only the outcomes which fit the specification of what you want to find the probability of, (call this count '*s*' -for 'specified').

The required probability is $\frac{s}{t}$ (but note : this is true only if each of the possible outcomes is equally likely.)

For the above case, the complete list of the 4 possible outcomes is:

H H

H T *

T H *

T T

There are 2 fitting the 'one head, one tail' specification and are marked *.

Thus the probability is $\frac{2}{4}$ (=1/2).

Note:

If, instead of tossing two coins together, one coin is tossed twice, the probability is the same. (Each possible outcome here refers to a possible combination of outcomes of the 1st and 2nd throw.) However, in this case, if the specification were '1st toss T, 2nd toss H', there is only one such possibility. so its probability is

$$\frac{1}{4}.$$

('Probability' of '1' means 'certainty'. One of life's few certainties is that a text on probability would contain an example involving dice…),

So:

What is the probability that throwing a pair of dice will yield two 3s?

As before, you can list all the possible outcome combinations. There are many, so it is easier to think.

One dice can give 6 different numbers. For each of these, the other dice has 6 possibilities. In total, 6×6 possible outcomes exist ($t = 36$).

Only one of these is a pair of 3s (or any other equal pair, of course).

So the probability is $\frac{1}{36}$ (deduced by mathematical reasoning, and not by intuition!).

Finally, we will look at the example of the National Lottery. This is a simple way of illustrating how untrustworthy intuition is.

Would you be likely to fill in the numbers 1, 2, 3, 4, 5, 6 on a lottery line?

Look at it from a ball-bubbling-machine's point of view. Even if it could exercise preference, it doesn't even know what is written on the balls. So, any combination of outcomes is, inevitably, equally (un)likely to occur.

(The only difference that choosing 1, 2, 3, 4, 5, 6 makes, is that, if you win with these numbers, it will make bigger headlines.).

Quite how weird probability can sometimes be is easy to demonstrate. Think about a piece of string, and choosing two points on it and cutting the string there. What is the probability that the pieces of string can make up as the sides of a triangle. It depends.

If you choose one point first, and cut the string; and then randomly choose another point in one of the two pieces of string (and which piece is chosen is itself random), then the probability is

$$\frac{1}{4}$$

However, if you choose your points at the same time and cut the string , the probability is

$$\frac{1}{6}.$$

Strange but true.